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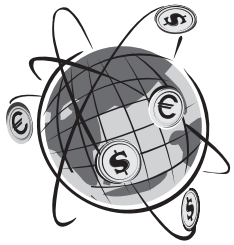

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ATTEMPT OF ESTIMATING CREDIT RISK OF SELECTED BANKS IN POLAND WITH USEOF RISK-NEUTRAL MODEL

Подано альтернативні підходи до оцінки кредитного ризику, у т.ч. аналіз умов оцінки вартості кредиту з урахуванням рівня кредитного ризику. Запропоновано використання ризик-нейтральної моделі для оцінки кредитного ризику приватних кредитів, представлено результати розрахунку кредитного ризику споживчих кредитів для окремих фінансових інститутів.

Ключові слова: кредитний ризик, споживчий кредит, оцінка кредитного ризику, ризик-нейтральна модель

This paper presents alternative approach to the issue of credit risk assessment, including the analysis of credit price conditions supposed to implicate the level of credit risk. The paper is concentrated on implementation of the risk-neutral model in order to assess credit risk of retail products, includes results of calculations carried out for selected financial institutions offering retail loans.

Keywords: credit risk, retail loans, credit risk assessment, risk-neutral model.

One of key elements of conducting investment activities is the issue of asset pricing. Cash flows generated by a given financial instrument constitute usually main source of its value, thus, their determination is so important. The uncertainty concerning future cash flows forces investors to estimate the probability of their occurrence, as well as to construct other measures enabling determination of real value of a given instrument. The acceptance of the efficient market hypothesis formulated by Louis Bachelier in 1900 (Bachelier 1990), revived by Eugene Fama (Fama 1970) seventy years later, generates the series of implications allowing for reversing traditional pricing process. Let us assume figuratively that instrument's market price is a risk function in the following form:

$$P = f(R),$$

Where P means instrument's market price, R is a measure of the uncertainty of cash flows generated by a financial instrument, *f()* is a function describing the relation between two above-mentioned elements. Due to the above-determined relation, it is possible to pose a question not about a price, but about the uncertainty accompanying generated cash flows. Assuming that the efficient market is able to set the P price properly, designation of a scale of the uncertainty inseparably connected with a given instrument may be written down symbolically in the following form:

$$R = f^{-1}(P).$$

Practitioners have used this approach so far usually in order to determine the insolvency risk of an issuer of bonds based on their prices. Deriving from it, the conception of risk neutrality enabled the analysis of insolvency risk in the context of spread understood as a difference between bond interest rate and return rate on risk-free instruments. Forecasts of the probability of insolvency set in this way do not always correspond with empirically observed values. As the first one, Arrow (1953) was conducting works on the issue of relation between so-called natural and neutral coefficients. That theory was constantly being developed in following years. For the first time, it was used in the context of derivative instrument pricing by Cox and Ross (1976). Then, Harrison and Kreps (1979), as well as Harrison and Pliska (1981), contributed to the development and popularisa-

tion of that theory. Promising results of researches may be found in works by Elton (2001), who conducted detailed research of spreads in the context of historical default rates and recovery rates. Delianedis and Geske (1998) made an attempt of explaining relations between observed default rates and indicators set according to the risk-neutrality principle.

The issue of estimating the insolvency risk of companies based on spreads is broadly described in the literature. Easy access to data in the form of historical default rates, as well as of ratings given by specialised companies, is favourable for publications. Publicly accessible bond quotations enabling determination of spreads are equally important for the development of this approach. Moreover, for more than twenty years the bond market has been dynamically developing. According to the Bank of International Settlements data, total value of bonds in the beginning of year 2010 exceeded 91 trillion dollars. All these cause that market efficiency in this sector is constantly getting stronger. Still consolidating assumptions accepted in the model cause that credit risk analysts commonly apply this methodology with some changes. One of the most spectacular applications is the Loan Analysis System (KPMG 1998) based on this theory and used by investment companies.

In this paper, one presents the attempt of using the conception of assessing insolvency risk in the context of retail credits, with keeping the risk-neutral approach. The aim of this paper is to establish the insolvency risk in the context of quoted method for the biggest retail banks in Poland. This aim requires changes in the classical model, in order to make it consistent with the structure of instalment retail products. In this paper, one verifies the hypothesis about the possibility of using the method for assessing credit risk of retail loans. Moreover, one discusses the usefulness of the method and interpretation of its results in the context of ignoring or not fulfilling some assumptions.

This paper starts from presentation of the risk-neutral valuation method. Then there is presented the pricing model, which takes into regard the specificity of retail loans. Further, one presents the example, in which credit price parameters offered by particular banks are analysed, in order to estimate the level of credit risk. The review of loan offers of particular banks enabled formulation of conclusions concerning the strategy implemented by banks.

Risk-assessment model presented in this paper is based on the risk-neutral valuation method. The above-mentioned approach states that investors having the possibility of trading assets on the financial market perform their pricing with use of risk-free instruments. One shall emphasise here that investors assess so-called expected profitability, that is the one resulting from cash flows generated by financial instruments, not from scheduled cash flows. Real cash flows, in the case of instruments like credits or loans, constitute capital and interest payments decreased (realigned) by possible losses on insolvency, as well as enriched by debt-collecting actions.

Direct implication of accepted assumptions is the conclusion that in the case of assets such as credits, so called bank margin, calculated as a difference between the effective interest rate and the rate specific for risk-free assets, is a “shock absorber” of losses, which source is credit risk.

The fundamental assumption here is the lack of arbitrage opportunity. This conception constitutes the basis of many theories commonly applied in the financial sector. It refers to all participants of the financial market. The following example constitutes figurative presentation of the significance of this principle. Let us assume that the lender notices that they might grant a loan (at a certain level of risk) with higher interest rate than other risk-free instruments on the market. Thus, theoretically higher interest rate implicates higher return rate, what is obviously favourable for a creditor. Nevertheless, this “surplus” of an interest rate above the profitability of risk-free instruments according to the presented theory has its source directly in future losses caused by the insolvency risk. Therefore, this “surplus” reflects losses connected with the credit risk.

A difference between lending interest rate and risk-free instrument interest rate does not mean that such a loss is going to be observed in the case of given loan. It means only that when having relatively big portfolio of such instruments, one shall expect an average loss on this actual level.

Let us assume as an example that the efficient profitability of average loan is on the higher level than the profitability of risk-free instruments. Observed lack of balance enables conducting arbitral transaction. In the case of "increased" profitability of loans, it is not hard to notice that if such a situation happened on the market, investors using that state of affairs would appear. Their activities would include investing funds in that asset, with simultaneous financing itself by lower, market cost. Difference between expected return rate on a risk asset and a cost of finance acquiring would lead to getting profits without a risk. Such an activity is purely arbitral. In the opposite case, that is when expected return rate on risky loans is lower than the rate characterising risk-free instruments, it would also force arbitral activities. As one mentioned before, one of fundamental assumptions of principles concerning financial instrument pricing is the principle of the lack of arbitrage opportunity. Thus, it turns out that the market is balanced if there is no arbitrage opportunity. Therefore, there is no possibility of appearance of implied probabilities of insolvency that would not reflect the real credit risk level.

Presented theory is in some way an alternative approach in relation to traditional methods of calculating the insolvency risk. Mostly the direct way of estimating risk is based on historical data allowing for calculation of so-called natural risk coefficients. Presented model consistent with the risk-neutrality principle focuses on estimating implied risk coefficients.

In order to present the idea of pricing according to the risk-neutral approach, let us assume that on the market one quotes risk-free bonds with any face value. Moreover, the access to these bonds is unlimited, and they can be continuously rolled. The return rate on such bonds is a risk-free rate. On the other hand, on the financial market other assets are accessible, which also generate cash flows, but there is no certainty concerning their realisation. Thus, investor has the opportunity of investing their funds in a risk-free instrument and an instrument with significant risk. In the case of risk-free bond, after one year investor receives the amount of investment increased by r, where r is a risk-free rate. Investor is sure about this occurrence due to accepted assumptions. After one year, value of a risky asset is a random variable taking two possible states:

$$\hat{X} = \begin{cases} RX & \text{with probability. } q \\ X & \text{with probability } (1\text{-}q) \end{cases},$$

Where X is a receivables resulting from the obligation assigned to a risky asset, q means risk-neutral probability of default, R is a recovery rate.

Expected value of the above-mentioned instrument is set in accordance with the following formula:

$$E(X) = qRX + (1-q)X \quad [1]$$

Thus, due to assumptions resulting from the presented approach, present value of a risky asset shall be equal to its expected value discounted with a risk-free rate:

$$\frac{X}{(1+s)} = \frac{E(X)}{(1+r)}.[2]$$

Where s is a interest rate assigned to a risk asset.

Thus, we receive

$$q = \frac{1}{1-R}\left[1 - \left(\frac{1+s}{1+r}\right)^{-1}\right][3]$$

The above-mentioned formula presents relation among an interest rate of a risky asset, probability of default, risk-free interest rate and recovery rate.

In order to use the above-mentioned formula in the process of assessing risk of much more complicated instrument as an retail loan, one shall conduct its decomposition into particular components creating its cash flows. Each instalment credit may be presented as a set of zero coupon bonds

maturing cyclically one after another. Maturities of particular bonds in this approach correspond with payment due dates, so they include the principal and interest components. Accepting equal instalments in relation to loan, after its decomposition, we obtain a set of bonds with non-linear decreasing nominal values. The lack of linearity is a reflection of the proportion of principal in fixed credit instalment. It is worth to mention that obtained set of bonds does not actually constitute a classical portfolio. That is because we assume that in the moment of occurrence of client's insolvency, we treat all bonds reflecting non-matured credit instalments as defaulted (with the accuracy to the recovery rate). Thus, the chronology of maturity in created set of bonds is so important.
Another problem on the way of assessing credit risk of retail loans, based on risk-neutrality approach, is a difficulty of recognition of bank costs resulting from granting and servicing the loan. These costs include salaries of sales assistants, back-office employees, as well as costs resulting from rents or payments for development and maintenance of IT services. Further, the level of costs will be established on a flat rate basis as a gross percentage of loan. One will carry out the cost estimation separately for each business line including cash loans, car loas and mortgage loans.

Taking into regard the above-mentioned stipulations, one assumed that time scheme of coupon maturity of a risk-free instrument is similar to the loan structure. Such an approach prevents from the appearance of undesirable liquidity excesses on the side of risk-free instrument in relation to loan, due to what presentation of the risk-neutrality conception becomes simpler. From the practical point of view, when analysing risk of whole credit portfolio, it is not difficult to construct analogous risk-free instrument portfolio with similar structure of capital flow. Thus, the following formula presents the relation between flows resulting from risky asset such as loan and risk-free asset:

$$\sum_{i=1}^{n} \frac{\left[\left(H_i - \sum_{j=1}^{i} L_j\right) \cdot \frac{s}{12} - k_i - P_{R,i} + P_{A,i}\right]}{\left(1+\frac{r}{12}\right)^i} = \sum_{i=1}^{n} \frac{\left[S_i \cdot \frac{r}{12}\right] + P_{L,i}}{\left(1+\frac{r}{12}\right)^i} \qquad [4]$$

Where:

H_i - scheduled outstanding in *i* month of loan life.

L_j - decrease of working outstanding as a result of credit risk in *j* month of loan life.

s - annual interest rate of loan.

n – efficient length of loan life counted in months.

S_i - risk-free instrument outstanding in *i* month.

k_i - costs of granting and loan servicing in the *i* loan instalment.

$P_{R,i}$ - principal value in *i* instalment that did not go to bank due to credit risk.

$P_{A,i}$ - scheduled principal flows of a loan.

$P_{L,i}$ - scheduled principal flows of a risk-free asset.

Formula no. 4 presents the balance of flows received from the asset in the form of loan and of flows of the risk-free asset. Simplifying, we can accept a treasury bond as a risk-free asset, thereby overlooking the issue of securing costs of bond risk with CDSs (Credit Default Swap). From the point of view of estimating implied risk level, we are going to be interested in a value of final loss in relation to a value of granted loan, that is:

$$EL = \frac{\sum_{j=1}^{n} L_j}{H_1}$$

where:
EL - percentile indicator of expected loss.
EL indicator presents expressed in percents part of principal lent to a borrower, which was not repaid due to realisation of a credit risk.
Further, as a simplification we state that lowering the loan outstanding due to credit risk happens once in 1/3 loan life period, but costs of granting and loan servicing are distributed in a linear way during whole credit life. The first assumption means that the occurrence of credit risk happens on average in 1/3 credit life. This assumption corresponds with empirical observations made by author based on retail loan portfolio. The second assumption about the linearity in time of incurred loan servicing costs also does not constitute significant simplification in relation to empirical observations.
One of the most important model parameters is a lending rate. In this case, model shall not take into regard nominal interest rate resulting from a credit agreement. Nominal interest rate is calculated from the gross value of loan, including the principal paid to a borrower, as well as credited commissions and loan insurance premium. These two last elements may constitute together almost 20% gross value of loan. They do not show features of real flows in the moment of granting a loan, but they are elements of a bank profit and loss account, similarly to interests set according to nominal interest rate. Much more favourable is taking into regard the real annual interest rate in a model - bank is obliged to inform on it under the Consumer Credit Act. Many banks calculating RAIR (Real Annual Interest Rate) do not take into regard costs connected with insuring the rest of debt. So calculations of the RAIR are conducted independently regarding all costs, in order to keep the comparability of data from different banks.

Calculations of the expected value of a credit risk were conducted in accordance with presented risk-neutral model and with use of commonly accessible bank loan offers. One analysed loan offers referring to three types of bank products: cash loan, car loan and mortgage loan. For the needs of that analysis, one assumed that risk-free rate was equal to the WIBOR 6M rate coming to 4.5% a year as for the day of analysis. From the point of view of conducting necessary calculations, the significant parameter is the value of costs of granting and loan servicing. This sum contains mostly of a loan granting cost, which depends on the accepted sale structure. In the example based on experiences gained in the financial sector, one assumed that in the case of cash loans, cost of granting and loan servicing should oscillate around 10% gross value of granted loan. For car loans, costs of granting and loan servicing was accepted on the level of 15% gross loan value; for mortgage loans that cost came to 2%. The above-mentioned costs for particular credit products refer to the whole period of loan life, although their significant part is incurred in the initial phase of credit life. Table 1 presents the results of analysis conducted for cash loans being sold by leading financial institutions in Poland.

Table 1

Cash loans risk - comparative analysis.

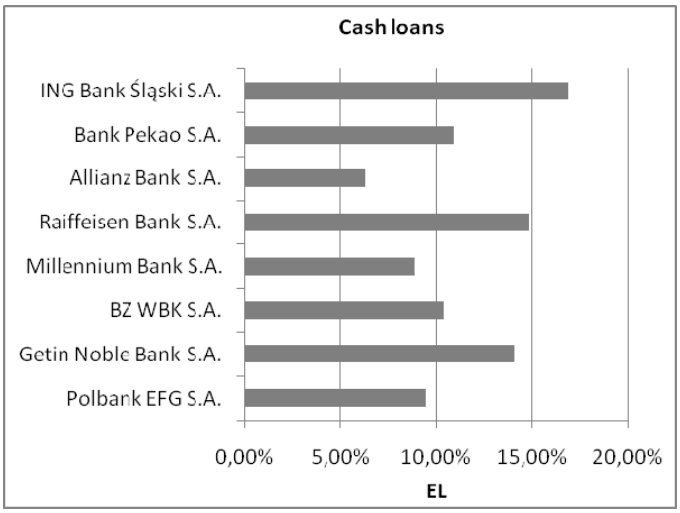
	Polbank EFG S.A.	Getin Noble Bank S.A.	BZ WBK S.A.	Millennium Bank S.A.	Raiffeisen Bank S.A.	Allianz Bank S.A.	Bank Pekao S.A.	ING Bank Śląski S.A.
Real Annual Interest Rate	22,9%	22,9%	19,5%	18,2%	23,6%	16,0%	20,0%	25,5%
cost of loan servicing	10,0%	10,0%	10,0%	10,0%	10,0%	10,0%	10,0%	10,0%
EL	9,51%	14,1%	10,4%	8,9%	14,9%	6,3%	10,9%	16,9%

Source: author's work.

The last row in Table 1 presents results of estimates of the EL indicator, which may be interpreted as an expected percentile loss of principal in a loan portfolio. Obtained results present small but clear differences of the level of estimated risk. Relatively highest value of EL was obtained by ING Bank Śląski S.A.

Reiffeisen Bank S.A. and Getin Noble S.A. got similar result. Decidedly lower values were obtained by other banks, among which Allianz Bank S.A. got the lowest risk level.

Figure 1 presents in a graphical way risk values obtained for particular financial institutions.



Source: author's work.

Fig. 1 Comparison of cash loan risk.

Table 2 presents data and results of risk analysis of particular financial institutions in the context of car loan portfolio.

Table 2

Car loan risk - comparative analysis

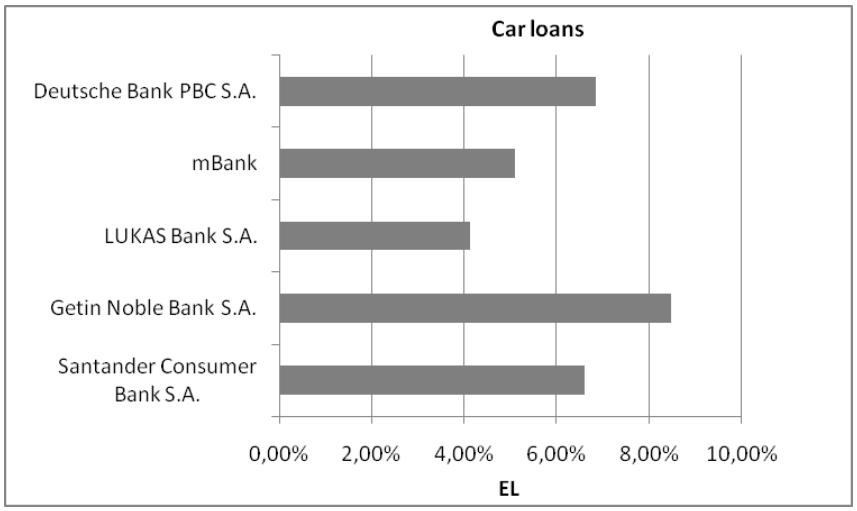
	Santander Consumer Bank S.A.	Getin Noble Bank S.A.	LUKAS Bank S.A.	mBank	Deutsche Bank PBC S.A.
Real Annual Interest Rate	15,0%	16,2%	13,5%	14,1%	15,2%
cost of loan servicing	15,0%	15,0%	15,0%	15,0%	15,0%
EL	6,63%	8,5%	4,1%	5,1%	6,8%

Source: author's work.

In the case of car loans, differences among losses are much lower than in the case of cash loans. The highest loss estimate was obtained for Getin Noble Bank S.A., which similarly to the case of previous loan product, shows the highest level of EL estimate. The lowest level of EL indicator is obtained for the Lukas Bank S.A.

Figure 2 presents in a graphical way the results of EL indicator estimates obtained for car loans.

In the case of car loans, one shall pay attention to relatively big convergence of obtained EL estimates. With average value of loss level coming to 6.24%, standard deviation was about 1.5%. It may be compared to EL standard deviation set for cash loans coming to 3.3%, that is over twice higher.



Source: author's work.

Fig. 2 Comparison of car loan risk.

Table 3 presents the analysis of an offer of mortgage loans directed to individuals buying residential real estates.

Table 3

Mortgage loan risk - comparative analysis

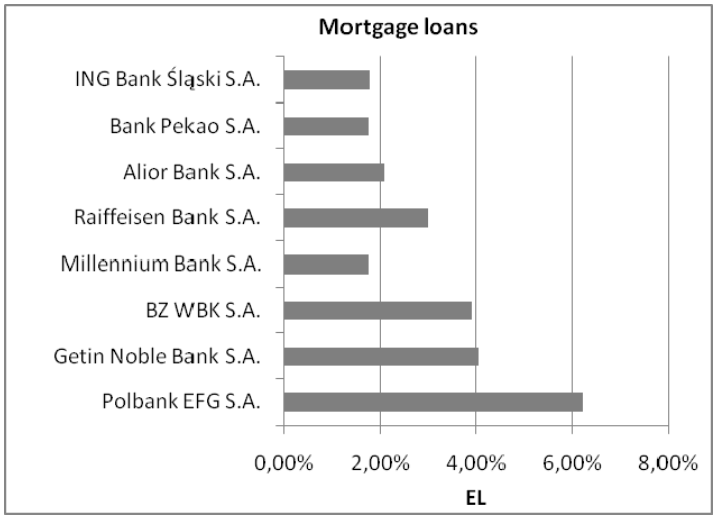
	Polbank EFG S.A.	Getin Noble Bank S.A.	BZ WBK S.A.	Millennium Bank S.A.	Raiffeisen Bank S.A.	Alior Bank S.A.	Bank Pekao S.A.	ING Bank Śląski S.A.
Real Annual Interest Rate	7,4%	6,7%	6,6%	5,9%	6,3%	6,1%	5,9%	5,9%
cost of loan servicing	2,0%	2,0%	2,0%	2,0%	2,0%	2,0%	2,0%	2,0%
EL	6,24%	4,07%	3,92%	1,76%	3,02%	2,10%	1,76%	1,79%

Source: author's work.

Relatively low interest rate of mortgage loans cause that obtained estimates of EL are close to each other. Mortgage loans have always been characterised by the lowest level of credit risk in relation to other credit products. One shall notice reasons of such a state of affairs both in a value of loan collateral, and in the fact that the aim of such loan concerns one of the most basic human needs. Thus, the borrower's determination to repay it is very high. Obtained results confirm low level of risk measured with use of EL estimator. Principal loss for distinguished banks comes on average to 3%. The highest risk level among all tested offers was obtained for Polbank, which level of EL came to 6.24%. Figure 3 presents in a graphical way results obtained for particular banks in relation to mortgage loans for purchase of residential real estates.

One shall treat results obtained during the analysis with great carefulness. Direct interpretation of EL indicator as a value of actual risk may lead to formulation of wrong conclusions. Use of models presented in this paper requires fulfilling rigorous assumption concerning the market, on which one conducts banking activities. Currently, market of banking activities in Poland and all over the world is significantly regulated.

High financial barrier of entering the market of banking activities, as well as significant element of gaining the know-how, cause that the assumption of model consisting of getting by investors one return rate regardless of a type of instrument they are investing in, may not be fulfilled. Then, direct result of this state of affairs can be overestimation of losses by a model.



Source: author's work.

Fig. 3 Comparison of mortgage loan risk.

Regardless of fulfilling the above-mentioned assumption, separate aspect is use of the model in order to compare and classify banks due to a level of risk . It appears that on the assumption of equal (or similar) ROA indicator (Return on Assets) for all banks, it is possible to order banks due to portfolio risk, even when obtained risk results are overestimated. Return on assets kept on the same level in different banks means that, after taking costs into regard (including risk costs), obtained financial results stay equal. Thus, differences in incomes resulting from different lending interest rates, as well as from other price parameters, must be balanced by incurred costs. In the opposite case, results measured by ROA would be different. Taking into account the structure of costs incurred by banks in Poland and similar costs of granting and loan servicing, the most important element of balancing incomes and costs enabling keeping equal for all ROA is a cost in the form of risk. Thus, the fundamental issue is verification of the assumption concerning the level of dispersion of return on assets obtained in Polish banking sector. Table 4 presents results of return on assets of selected banks for year 2010.

Results presented in Table 4 are characterised with relatively high similarity. Average value for banks came to 1.19%, and most of indicators are placed within the range from 1% to 2%.

Table 4

ROA indicators of selected banks for year 2010

	Getin Noble Bank S.A.	BZ WBK S.A.	Millennium Bank S.A.	Raiffeisen Bank S.A.	BRE Bank S.A.	Bank Pekao S.A.	ING Bank Śląski S.A.	Santander Consumer Bank S.A.	Bank Handlowy
ROA	1,15%	1,77%	0,71%	1,05%	0,74%	1,90%	1,21%	0,26%	1,92%

Source: author's work.

The exception here includes BRE Bank, Santander Consumer Bank and Millennium Bank. Similar results of ROA indicators of different banks indicate obtaining by them similar profitability of loans. Therefore, this constitutes strong argument supporting the thesis about the occurrence of a dependency between price parameters and credit risk. When we assume that bank profitability is approximately the same, higher risk in one bank has to be compensated by higher level of profits from credit in the form of credit interest or other elements with the character of commission. Some disturbances of this relation may be caused by other bank costs, as for example deposit costs. Nevertheless, the deposit market in Poland is so stable, and differences in deposit interest rates so small

that it can be omitted. Thus, the assumption stating that costs of funding in banks are approximately the same seems to be justified. In addition, the assumption of similar structure of other costs in banks seems to be reasonable simplification.

Undoubtedly, information concerning return on assets regarding particular business lines including separately cash loans, car loans and mortgage loans, would have favourable influence on the quality of estimates. However, thanks to interviewing banks usually protecting this kind of information, one managed to acquire the knowledge that differences in the profitability level of particular products are not so significant that they could significantly influence final estimate of a portfolio risk.

The above-presented aspects conditioning the possibility of using described approach to assessing credit risk highlight the problem of interpreting obtained results. The lack of meeting the above-mentioned assumptions may mean underestimation or overestimation of a value of real risk. What's more, risk estimations obtained due to risk neutral approach gets one more interpretation that is very significant from the practical point of view. Releasing this model from the assumption stating that bank owners obtain return rate on capital analogous to return rate on risk-free instruments, we receive risk estimate with interesting interpretation. One shall point out here that it is not important now if bank is managed in order to make return rate higher than risk-free rate, or if bank is managed in the way causing capital depletion resulting from incurred losses. Therefore, the value of estimated risk in the form of EL reflects maximum loss, at which bank earns on loan portfolio as much as it would earn on investing the same funds in risk-free instruments. Moreover, when we replace risk-free rate with average interest rate of bank's deposit, then estimated EL informs us on maximum risk level, exceeding of which may cause negative portfolio profitability. In this way, the estimation of EL gets new interpretation in the form of threshold level of risk conditioning positive financial result. The value of EL set for Allianz Bank for cash loans may be an interesting example illustrating this risk threshold. This bank with the result of 6.3% is characterised by the lowest EL value among all analysed banks. It is worth to add that average EL value for analysed banks came to 11.6%, which is almost two times higher than in Allianz Bank. One shall mention here the significant increase of credit risk in all retail banks because of financial crisis in year 2008. Thereby, buffer set for credit risk in the form of maximum risk increase conditioning positive financial result has been narrowed down, and in many cases, it could even be exceeded. This situation will happen the soonest in banks with the lowest EL value estimated in accordance with the risk-neutral approach. Keeping then positive bank profitability would require either the extraordinary policy limiting costs of activities, or much more efficient risk management system than in other banks. Eventually, it seems that border risk value for Allianz Bank was exceeded, what may be proved by financial results from year 2009 on the level of 126 million PLN losses and announced losses in year 2010 exceeding 100 million PLN.

Undoubtedly, the model of credit risk assessment used in this paper has many disadvantages, apart from its advantages. One of basic inconveniences is the fundamental assumption about the risk-neutrality of investors, resulting in obtaining set return rate for all investments. In the most economically developed countries, such an assumption seems to be justified, but in Poland bank service market is still at the initial stage of development. The best example of this is great diversity of prices of credit offers. Nevertheless, results of credit risk of selected banks obtained in the course of analysis oscillate around the risk value observed upon historical data. Thereby, this confirms the usefulness of the model and its utility in the process of credit risk estimation. Certainly, obtained results motivate to further development of this type of models. On the one hand, they are the alternative for traditional models of credit risk measurement; on the other hand, they often constitute the only possibility of risk assessment when there is no access to historical data. Exactly this feature may be particularly useful and may constitute support of the risk management system in a

bank, which based on the described model may carry out periodical positioning of its risk level in the background of other banks. When estimating risk of its competitors, bank is able to control market situation and thereby assess the efficiency of its risk management policy.

Another application of presented model is using it to establish admissible credit risk level enabling the maintenance of positive portfolio profitability. Thus, persons responsible for the risk management process in bank may follow obtained results in the process of establishing the cut-off level in a scoring model. The usefulness of the model is greater here, because it does not require inconvenient assumption about the risk-neutrality of investors.

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